Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1(a)	$-3 < x < 1 \quad x > 5$	**B1**	
1(b)	$-\frac{1}{3}(x+3)(x-1)(x-5)$	3	**B1** for a negative cubic function **B1** for a cubic function multiplied by $\frac{1}{3}$ **B1** for $(x+3)(x-1)(x-5)$
2(a)	$a = \frac{10}{3}$ or $3\frac{1}{3}$	**B1**	
	$b = \frac{7}{3}$ or $2\frac{1}{3}$	**B1**	
	$c = \frac{9}{2}$ or $4\frac{1}{2}$ or 4.5	**B1**	
2(b)	$10(2^p)^2 - 17(2^p) + 3 = 0$ $(5(2^p)-1)(2(2^p)-3) = 0$ $2^p = \frac{1}{5},\ 2^p = \frac{3}{2}$	**M1**	For recognition of a quadratic in 2^p, attempt to factorise and solve for 2^p
	$p = \frac{\ln\frac{1}{5}}{\ln 2}$ or $p = \frac{\ln 1.5}{\ln 2}$ oe	**M1**	For correct attempt to deal with $2^p = k$
	-2.32	**A1**	
	0.585	**A1**	
3(a)	$\lg\frac{1000a^2}{b^4}$	4	**B1** for $3 = \lg 1000$
			B1 for use of power rule once
			B1 for use of addition or subtraction rule once
			B1 All correct

Question	Answer	Marks	Guidance
3(b)	**Either** $3\log_a 4 = \frac{3}{\log_4 a}$	B1	
	$2(\log_4 a)^2 - 7\log_4 a + 3 = 0$ $(2\log_4 a - 1)(\log_4 a - 3) = 0$ $\log_4 a = \frac{1}{2}$ or $\log_4 a = 3$	M1	For obtaining a quadratic equation and solution
	$a = 4^{\frac{1}{2}}$ or $a = 4^3$	M1	**Dep** For dealing with the logarithm correctly once, may be implied by a correct solution
	64	A1	
	2	A1	
	Or $2\log_4 a = \frac{2}{\log_a 4}$	(B1)	
	$3(\log_a 4)^2 - 7\log_a 4 + 2 = 0$ $(3\log_a 4 - 1)(\log_a 4 - 2) = 0$ $\log_a 4 = \frac{1}{3}$ or $\log_a 4 = 2$	(M1)	For obtaining a quadratic equation and solution
	$a^{\frac{1}{3}} = 4$ or $a^2 = 4$	(M1)	**Dep** For dealing with the logarithm correctly once, may be implied by a correct solution
	64	(A1)	
	2	(A1)	
4	$\tan\left(2x + \frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$	B1	
	$x = -\frac{7\pi}{12},\ -\frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{11\pi}{12}$	3	**M1** for using correct order of operations **A1** for two correct solutions **A1** for two further correct solutions and no other solutions in range

Question	Answer	Marks	Guidance
5	**Either** Maximum when $\sin\frac{x}{3}=1$ or minimum when $\sin\frac{x}{3}=-1$	**M1**	For recognition that value of maximum or minimum is necessary
	$c=9$	**A1**	
	$c=-1$	**A1**	
	or $\frac{dy}{dx}=\frac{5}{3}\cos\frac{x}{3}$ When $\frac{dy}{dx}=0$, $\sin\frac{x}{3}=+1$ or -1	**(M1)**	For differentiation, equating to zero to obtain values for $\sin\frac{x}{3}$
	$c=9$	**(A1)**	
	$c=-1$	**(A1)**	
6(a)	$0=-\frac{5}{4}+\frac{a}{4}+5+b$	**M1**	For use of the factor theorem
	$-24=-10+a+10+b$	**M1**	For use of the remainder theorem
	$a+4b=-15$ $a+b=-24$ leading to	**M1**	**Dep** on both previous **M** marks for solution of *their* equations without using a calculator
	$a=-27$, $b=3$	**A1**	
6(b)	$(2x+1)(5x^2 \text{..........} + \textit{their}\ \ b)$	**M1**	Allow for observation or algebraic long division. *Their a* and *b* must be integers.
	$(2x+1)(5x^2-16x+3)$	**A1**	
	$(2x+1)(5x-1)(x-3)$	**2**	**M1** for attempt to factorise *their* 3-term quadratic **A1** all correct from fully correct working
6(c)	3	**B1**	**FT** on *their* (integer) *b*
7(a)(i)	**b** – **a**	**B1**	
7(a)(ii)	**c** – **b**	**B1**	

Question	Answer	Marks	Guidance
7(a)(iii)	$n\overrightarrow{AB} = m\overrightarrow{BC}$	M1	For substitution of *their* **(i)** and **(ii)** into $n\overrightarrow{AB} = m\overrightarrow{BC}$
	$n\mathbf{a} + m\mathbf{c} = (m+n)\mathbf{b}$	A1	For correct manipulation to obtain the **given answer**
7(b)	$2\lambda - 4\mu + 4 = 4\lambda + 4$ or $\lambda + 7\mu - 7 = -2\lambda - 2$	M1	For equating like components at least once, allow unsimplified
		M1	**Dep** for solving *their* equations to obtain both λ and μ
	$\mu = 5$	A1	
	$\lambda = -10$	A1	
8(a)	**Either** Starting with a 6: 120 ways	B1	May be implied by final answer
	Starting with 5, 7 or 9: 540 ways	B1	May be implied by final answer
	Total 660	B1	
	Or Alternative 1 Ending with a 6: 180 ways	(B1)	May be implied by final answer
	Ending with 0 or 4: 480ways	(B1)	May be implied by final answer
	Total 660	(B1)	
	Or Alternative 2 11 ways of obtaining even 5-digit numbers which start with 5, 6, 7, 9	(B1)	For $11 \times k$ May be implied by final answer
	${}^5P_3$ ways of arranging remaining 3 digits: 60	(B1)	For $m \times 60$ where m is from an attempt to list all cases for first and last digits May be implied by final answer
	$11 \times 60 = 660$	(B1)	
	Or Alternative 3 Total arrangements ${}^7P_5$ minus (all odds + evens starting with 1 + evens starting with 0 or 4) = 2520 – (1440 + 180 + 240)	(B2)	For 2520 – (1440 + 180 + 240)
	660	(B1)	

Question	Answer	Marks	Guidance
8(b)	$\frac{n!}{(n-4)!4!}=\frac{6n!}{(n-2)!2!}$	B1	
	$(n-2)(n-3)=72$	2	**B1** for $(n-2)(n-3)$
			B1 for 72
	$n=11$ only	2	**M1** for correct attempt to form and solve a quadratic equation **A1** for $n=11$ only
9(a)	$AOD=2\times\tan^{-1}\left(\frac{2}{3}\right)$	M1	For correct method to find *AOD*
	$AOD=1.1760...$ $AOD=1.176$ [to 3dp]	A1	Need to see 4 dp or more to justify 3 dp answer
9(b)	Major arc $MN=(2\pi-1.176)12$	B1	
	ND or *MA* $=12-\sqrt{13}$	B1	
	Perimeter = major arc *MN* + *MA* + *ND* + 16 oe	B1	For *their* values in a correct plan, may be implied by a correct answer
	Perimeter = 94.1	B1	
9(c)	Minor sector area = $\frac{1}{2}\times1.176\times12^2$ **or** Major sector area = $\frac{1}{2}\times(2\pi-1.176)\times12^2$	B1	
	Area = major sector area – remainder of rectangle or Area = area of circle – minor sector area – remainder of rectangle or Area = circle – rectangle – minor sector + triangle *AOD*	B1	For *their* values in a correct plan, may be implied by a correct answer
	Area = 350	B1	Allow greater accuracy
10(a)	At *A* $y=4$	B1	
	At *B* $y=\frac{13}{16}$ or 0.8125	B1	

Question	Answer	Marks	Guidance
10(b)	**Either** Area of trapezium = $\frac{231}{32}$	**B1**	Allow unsimplified
	$\int_{-1}^{2}\frac{1}{(x+2)^2}+\frac{3}{x+2}\,dx$ $=\left[-\frac{1}{x+2}+3\ln(x+2)\right]_{-1}^{2}$	**2**	**B1** for $-\frac{1}{x+2}$ **B1** for $3\ln(x+2)$
	$\left[\left(-\frac{1}{4}+3\ln 4\right)-(-1)\right]$	**M1**	For correct use of limits in *their* integral, but must have at least one of the two preceding **B** marks
	Area = $\frac{207}{32}-\ln 64$	**2**	**A1** for $\frac{207}{32}$ **A1** for $-\ln 64$
	Or $\int_{-1}^{2}-\frac{17}{16}x+\frac{47}{16}-\frac{1}{(x+2)^2}-\frac{3}{x+2}\,dx$ $\left[\left(-\frac{17}{32}x^2+\frac{47}{16}x+\frac{1}{x+2}-3\ln(x+2)\right)\right]_{-1}^{2}$	**(3)**	**B1** for $-\frac{17}{32}x^2+\frac{47}{16}x$ **B1** for $\int\frac{1}{(x+2)^2}dx=-\frac{1}{(x+2)}$ **B1** for $\int\frac{3}{x+2}dx=3\ln(x+2)$
	$\left(-\frac{17}{8}+\frac{47}{8}+\frac{1}{4}-3\ln 4\right)-\left(-\frac{17}{32}-\frac{47}{16}+1\right)$	**(M1)**	For correct use of limits in *their* integral, but must have at least one of the two preceding **B** marks
	Area = $\frac{207}{32}-\ln 64$	**(2)**	**A1** for $\frac{207}{32}$ **A1** for $-\ln 64$
11(a)(i)	0	**B1**	
11(a)(ii)	–3	**B1**	
11(a)(iii)	$\left(\frac{1}{2}(25+15)\times 30\right)+\left(\frac{1}{2}(30+60)\times 10\right)+\left(\frac{1}{2}\times 20\times 60\right)$	**M1**	For an unsimplified expression for the required area allowing at most one incorrect length
	Total distance = 1650	**A1**	
11(b)(i)	$v=4\cos\frac{5\pi}{3}-4$ $=-2$	**M1**	
	Speed = 2	**A1**	

Question	Answer	Marks	Guidance
11(b)(ii)	$a = -12\sin 3t$	**B1**	
	$\sin 3t = 0$ $3t = \pi$ Leading to	**M1**	For equating to zero and attempt to solve to obtain t, allow if in degrees
	$t = \frac{\pi}{3}$	**A1**	
11(b)(iii)	$s = k\sin 3t - 4t\,(+c)$	**M1**	
	$s = \frac{4}{3}\sin 3t - 4t$	**A1**	